Productivity as a Relation of Economy Output and Input.
Role of Productivity in Price Dynamics.

Abstract

This paper investigates the relationship between output goods in an economy and the input factors utilized to produce them. By defining stable prices relative to a chosen base good (or basket of goods), it illustrates how inflation or deflation can be controlled. The concept of productivity plays a key role in determining changes in relative prices: when the productivity of an individual good or service grows faster or slower than the average productivity, its price decreases or increases relative to others. Likewise, in the input (factor) economy, the real price of a factor changes in line with its productivity. Through these analyses, the paper shows that both GDP (output) and GDI (input) can be linked via an appropriate choice of base prices, helping to maintain a stable price environment and providing a consistent way to measure real growth over time.

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1 Introduction and Overview

All goods in an economy have relationships with each other at any moment in time. We know how many loaves of bread can be exchanged for 1 lb. of apples, how many gallons of milk can be given for 1 loaf of bread, how many dozens of eggs can be given for 1 gallon of milk, and how many gallons of milk for 1 oz. of gold. If we have n goods, then the total number of relationships would be $\sum_{i=1}^{n-1} i$. However, all relationships are dependent and can be derived from others. If we know a relationship between milk and eggs, and between milk and bread, then we know a relationship between eggs and bread. The actual number of meaningful relationships is $n-1$. If we have 5 different goods in our economy, then the total number of relationships would be $\sum_{i=1}^{5-1} i = 10$, and the meaningful number of relationships would be $(5-1) = 4$.

We can choose any good as a base and specify relationships to other goods. In that case, we cover all relationships between goods in our system. If we choose 1 loaf of bread as a base and specify relationships to other goods, then we describe our system.

After we specify relationships, we can specify prices. We'll just set any value to the base product (say, 1). Then all other products will assume values because of existing relationships. To make the following statements easier to comprehend, we'll mark those relations as relations between the utilities those goods have (as we are analyzing a consumption system). If 1 dozen of eggs exchanges for 0.5 loaves of bread, that means 1 dozen of eggs has 0.5 times less utility than 1 loaf of bread. So, we can assume our prices as an expression of corresponding utilities. If we mark 1 loaf of bread as 1 utility in a base period, for example, then we'll remember that forever in our system. (No, we won't remember that 1 loaf of bread is always 1 utility; we'll remember that 1 loaf of bread *in a base period* is 1 utility.) If you don't like using utilities, you can imagine dollars. Using dollars (with a fixed deflator always equal to 1) will not change the following concepts.

2 Base Year Setting for the System

Let's have an economy that produces n final goods during the base year and year i. We have the same product types each year (for the sake of simplicity; it will not be difficult to handle some products being added or removed each year). We'll call it the *output economy*. Each product j is produced in the amount N_{ij}. We know all exchange rates between goods. We can express all relations based on some product, say a, in the base year. We'll mark them as r_{0j}^{a}. (We can imagine apples or gold as base products, for example.) So, we can set a price for 1 unit of product a as 1 and derive all prices from a to other products. Thus, the prices become $p_{0j} = r_{0j}^{a}$. Let's call the sum $\sum_{j=0}^{n} N_{0j}p_{0j}$ the base year GDP (GDP_0).

We call it GDP because it resembles Gross Domestic Product (GDP) used as a measure of a country's economic activity. However, it is not exactly the same, as it doesn't take into account exports (imports), government consumption expenditures, etc. (again, for simplicity).

There is not much that is interesting about GDP_0. We are more interested in the economy's dynamics and how to evaluate it in subsequent years. So, let's set next year's prices again based on the relation to product a. The relations will be r_{ij}^a. However, we cannot set the price of product a in year i as 1.

$$\sum_{j=0}^{n}(N_{ij}p_{ij}) = \sum_{j=0}^{n}(N_{ij}p_{0j})$$

We will not spend time explaining why stable prices are important—just note that both inflation and deflation are bad for the economy. If

$$\sum_{j=0}^{n}(N_{ij}p_{ij}) > \sum_{j=0}^{n}(N_{ij}p_{0j}),$$

we have inflation. Conversely, if

$$\sum_{j=0}^{n}(N_{ij}p_{ij}) < \sum_{j=0}^{n}(N_{ij}p_{0j}),$$

we have deflation.

$$\sum_{j=0}^{n}(N_{ij}p_{ia}r_{ij}^a) = \sum_{j=0}^{n}(N_{ij}p_{0j})$$

$$p_{ia}\sum_{j=0}^{n}(N_{ij}r_{ij}^a) = \sum_{j=0}^{n}(N_{ij}p_{0j})$$

$$\sum_{j=0}^{n}(N_{ij}p_{0j}) = \sum_{j=0}^{n}(p_{0a}N_{ij}r_{0j}^a) = p_{0a}\sum_{j=0}^{n}(N_{ij}r_{0j}^a)$$

$$p_{0a}\sum_{j=0}^{n}(N_{ij}r_{0j}^a) = p_{ia}\sum_{j=0}^{n}(N_{ij}r_{ij}^a)$$

$$\frac{p_{ia}}{p_{0a}} = \frac{\sum_{j=0}^{n}(N_{ij}r_{0j}^a)}{\sum_{j=0}^{n}(N_{ij}r_{ij}^a)}$$

$$\frac{RGDP_i}{GDP_0} = G_i^{out}$$

$$\frac{p_{ia}}{p_{0a}} = \frac{G_i^{out}\sum_{j=0}^{n}(N_{0j}r_{0j}^a)}{\sum_{j=0}^{n}(N_{ij}r_{ij}^a)}$$

So, the price change of product a can be expressed as a relation of utility growth in year i to the average growth of relative utility of all products compared to product a.

3 Introducing Production Factors (Input Economy)

We are now ready for the next part of our research.

Let's introduce production factors into our system (the *input economy*). Examples of input factors are labor, commodities like oil or metals, return on capital (profits), rent, interest, etc.

We assume that our economy uses m production factors. For simplicity, we'll assume m is the same each year and that all production factors are the same. (Chaining will allow the system to be more realistic, though we do not need it for this research.) Each production factor j is used in the amount M_{ij} in year i. We know all exchange rates between production factors. We can express all relations based on some factor, say b, in the base year. We'll mark them as q^b_{0j}.

Let's first consider the input economy to be independent, the same way we considered the output economy.

We expect the relation of the base factor's prices in different periods to be (as in the output economy):

$$\frac{c_{ib}}{c_{0b}} = \frac{\sum_{j=0}^{m}(M_{ij}q^b_{0j})}{\sum_{j=0}^{m}(M_{ij}q^b_{ij})} = \frac{G_i^{in}\sum_{j=0}^{m}(M_{0j}q^b_{0j})}{\sum_{j=0}^{M}(M_{ij}q^b_{ij})}$$

$$f_i^{ind} = \frac{G_i^{in}\sum_{j=0}^{m}(M_{0j}q^b_{0j})}{\sum_{j=0}^{M}(M_{ij}q^b_{ij})}$$

We found this relation of prices under the assumption that the input system is independent. However, that assumption is not correct. It is dependent on the output system, and this dependence can be expressed by the equation $GDI_i = GDP_i$, where

$$GDI_i = \sum_{j=0}^{m}(M_{ij}c_{ij}).$$

We call it GDI because it resembles the GDI used as a measure of a country's economic activity based on total income generated within a country. However, it is not exactly the same, as it doesn't take into account taxes, subsidies, etc. (again, for simplicity).

$$\sum_{j=0}^{n}(N_{0j}p_{0j}) = \sum_{j=0}^{m}(M_{0j}c_{0j}) = \sum_{j=0}^{m}\left(M_{0j}c_{0b}q^b_{0j}\right) = c_{0b}\sum_{j=0}^{m}(M_{0j}q^b_{0j})$$

$$\frac{\sum_{j=0}^{n}(N_{0j}p_{0j})}{\sum_{j=0}^{m}(M_{0j}q^b_{0j})}$$

$$c_{ib} = \frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{m}(M_{ij}q^b_{ij})}$$

$$\frac{c_{ib}}{c_{0b}} = \frac{\frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{m}(M_{ij}q_{ij}^{b})}}{\frac{\sum_{j=0}^{n}(N_{0j}p_{0j})}{\sum_{j=0}^{m}(M_{0j}q_{0j}^{b})}} = \frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{n}(N_{0j}p_{0j})} \times \frac{\sum_{j=0}^{m}(M_{0j}q_{0j}^{b})}{\sum_{j=0}^{m}(M_{ij}q_{ij}^{b})} = f_i^{ind}\frac{G_i^{out}}{E_i} = f_i^{dep}$$

$$\frac{f_i^{dep}}{f_i^{ind}} = \frac{G_i^{out}}{E_i}$$

As we can see, prices in the input system change according to the relation of growth between output and input systems.

Thus, the stable prices equation for the input system will be:

$$\sum_{j=0}^{m}(M_{ij}c_{ij}) = \frac{G_i^{out}}{E_i}\sum_{j=0}^{m}(M_{ij}c_{0j})$$

We can call $G_i^{out} = G_i$.

$$\sum_{j=0}^{m}(M_{ij}c_{0j}) = BGDI_i,$$

where $BGDI_i$ is GDI in year i with base period prices.

$$\frac{G_i}{E_i}BGDI_i = O_i \times BGDI_i = RGDP_i$$

$$\frac{G_i}{E_i} = O_i,$$

where O_i is the growth of economy productivity in period i compared to the base period. This is the productivity growth from all input factors (not just labor).

We can set $O_i \times BGDI_i$ as $RGDI_i$ (Real GDI).

$$O_i \times BGDI_i = RGDI_i = RGDP_i$$

We see that GDI_i with base prices ($BGDI_i$) doesn't match the real GDI_i ($RGDI_i$), unlike the output system where GDP_i with base prices ($BGDP_i$) equals $RGDP_i$.

4 Choosing a Base and Keeping Deflator at 1

$$\frac{p_{ia}}{p_{0a}} = \frac{\sum_{j=0}^{n}(N_{ij}r_{0j}^{a})}{\sum_{j=0}^{n}(N_{ij}r_{ij}^{a})}.$$

What must we choose to ensure the deflator stays at 1?

If we choose p_{0a} to be based on GDP_0 and assign any number k to it, then the deflator will always be 1.

$$\sum_{j=0}^{n}(N_{0j}r^a_{0j}) = \sum_{j=0}^{n}(N_{0j}r^a_{ij}) = k$$

$$p_{ia} = p_{0a} = k$$

$$D^a_i = \frac{p_{ia}}{p_{0a}} \times \frac{\sum_{j=0}^{n}(N_{0j}r^a_{ij})}{\sum_{j=0}^{n}(N_{0j}r^a_{0j})} = 1$$

We can assume GDP_0 to have 1 utility (choosing 9.23456 would make no difference) and mark it as a price p_{ou} ($GDP_0 = p_{ou} = 1$ utility).

$$p_{0u} = p_{iu} = GDP_0 = 1$$

$$GDP_i = G_i \times GDP_0 = G_i \times p_{0u} = G_i$$

Note that we assumed for simplicity that the same n goods exist in each period. It is not an issue to have different goods or different numbers of goods. We can still make GDP_i compatible with GDP_0 by using chaining.

We can use similar reasoning for the input system (connected to output).

If we choose c_{0a} to be based on GDI_0 and assign any number k to it, then the deflator will always be O_i.

$$\frac{\frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{m}(M_{ij}q^b_{ij})}}{\frac{\sum_{j=0}^{n}(N_{0j}p_{0j})}{\sum_{j=0}^{m}(M_{0j}q^b_{0j})}} = \frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{n}(N_{0j}p_{0j})} \times \frac{\sum_{j=0}^{m}(M_{0j}q^b_{0j})}{\sum_{j=0}^{m}(M_{ij}q^b_{ij})} = \frac{kG_i}{kE_i} = O_i$$

$$c_{0b} = k$$
$$c_{ib} = kO_i$$

$$\frac{c_{ib}}{c_{0b}} = O_i$$

$$\frac{\frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{m}(M_{ij}q^b_{ij})}}{\frac{\sum_{j=0}^{n}(N_{0j}p_{0j})}{\sum_{j=0}^{m}(M_{0j}q^b_{0j})}} = \frac{\sum_{j=0}^{n}(N_{ij}p_{ij})}{\sum_{j=0}^{n}(N_{0j}p_{0j})} \times \frac{\sum_{j=0}^{m}(M_{0j}q^b_{0j})}{\sum_{j=0}^{m}(M_{ij}q^b_{ij})} = \frac{kG_i}{kE_i} = O_i$$

We can assume GDI_0 to have $C_0 = 1$ contribution (choosing 11.11111 would make no difference). Then

$$BGDI_i = E_i C_0 = E_i.$$

$$RGDI_i = RGDP_i = O_i \times BGDI_i = E_i O_i \times GDI_0 = G_i \times GDI_0 = G_i \times GDP_0$$

We can use GDP_0 (or any subset of GDP_0 that exactly represents the whole GDP_0) as a GDP standard to link a country's currency to it. For example, we could say that $1 is equal to 1/27.36 trillion of 2023 USA GDP and be ready to exchange a corresponding basket of goods for that $1.

5 Price Dynamics Under Stable Prices

Let's evaluate the dynamics of prices of different types of goods or production factors in an economy with stable prices.

First, let's see how the price changes for a basket representing the base period GDP (1 utility). As we saw before, it does not change:

$$p_{iu} = p_{0u} = \text{constant}.$$

A price for the basket of input factors representing the base period GDI (1 contribution) does change proportionally to productivity:

$$c_{ic} = O_i \times c_{0c}.$$

Let's see what prices for goods and services should be in year i compared to base year prices. We know the absolute cost of 1 contribution unit (c_{ic}) each year (in the base year, it can be expressed as c_{0c} = base year GDP = base year GDI). We can express the production cost of each product in contribution units l_{ij}.

$$p_{0j} = c_{0c} \times \frac{p_{0j}}{c_{0c}} = c_{0c} \times l_{0j}$$

$$p_{ij} = c_{ic} \times \frac{p_{ij}}{c_{ic}} = c_{ic} \times l_{ij}$$

$$\frac{p_{ij}}{p_{0j}} = \frac{c_{ic} \times l_{ij}}{c_{0c} \times l_{0j}} = \frac{c_{ic}}{c_{0c}} \times \frac{1}{\alpha^{\uparrow}_{ij}} = \frac{c_{0c} \times O_i}{c_{0c}} \times \frac{1}{\alpha^{\uparrow}_{ij}} = \frac{O_i}{\alpha^{\uparrow}_{ij}}$$

Where $\alpha^{\uparrow}_{ij}$ is the productivity change for good j in year i, compared to the base year.

$$p_{ij} = p_{0j} \times \frac{O_i}{\alpha^{\uparrow}_{ij}}$$

As we can see, the price for good (service) j will increase if the productivity change for that good is below the mean productivity change. It will not change if the productivity change for that good is the same as the mean. It will decrease if the productivity change for that good is higher than the mean.

We can denote $\frac{\alpha_{ij}^{\uparrow}}{O_i} = \widehat{\alpha_{ij}^{\uparrow}}$ as the increase in productivity of production factor j in period i relative to the average productivity growth:

$$p_{ij} = \frac{p_{0j}}{\widehat{\alpha_{ij}^{\uparrow}}}.$$

Let's see what prices for input factors should be in year i compared to base year prices.

$$c_{0j} = p_{0u} \times \frac{c_{0j}}{p_{0u}} = p_{0u} \times u_{0j}$$

where p_{0u} is the price of 1 utility in the base period, and u_{0j} is the amount of utility produced by 1 unit of input factor j in the base period (the productivity of input factor j in the base period).

The real price of 1 unit of input factor j (such as 1 hour of labor) in period i is

$$c_{ij} = p_{iu} \times u_{ij},$$

where p_{iu} is the real price of 1 utility in period i, and u_{ij} is the amount of utility produced by 1 unit of input factor j in period i (the productivity of input factor j in period i).

$$\frac{c_{ij}}{c_{0j}} = \frac{p_{iu}u_{ij}}{p_{0u}u_{0j}} = \frac{p_{iu}u_{0j}\beta_{ij}^{\uparrow}}{p_{0u}u_{0j}} = \frac{p_{iu}\beta_{ij}^{\uparrow}}{p_{0u}}$$

where $\beta_{ij}^{\uparrow}$ is the productivity change of input factor j in period i compared to the base period.

Since the price of 1 utility is constant,

$$c_{ij} = c_{0j}\,\beta_{ij}^{\uparrow}.$$

An input factor's price grows according to its productivity change. If the productivity of that factor did not change, then its price will not change.

6 Conclusion

In this paper, we explored how an economy's output of goods and its set of production factors interact under stable prices. By selecting a reference base for both output (such as one good or a basket of goods) and input factors, it becomes possible to analyze price movements strictly in terms of productivity changes. Goods whose productivity improves at a rate below the average productivity growth tend to increase in price, and those with higher-than-average productivity gains experience lower prices. Analogously, the costs of input factors shift in proportion to how their productivity changes. By carefully linking GDP and GDI via consistent choices of base prices, an economy can avoid inflation or deflation, thus maintaining stable pricing and a clear picture of real growth over time.